An Accelerated Inverse Perturbation Method for Structural Damage Identification

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In the previous study, the inverse perturbation method was used to identify structural damages. Because all unmeasured DOFs were considered as unknown variables, considerable computational effort was required to obtain reliable results. Thus, in the present study, a system condensation method is used to transform the unmeasured DOFs into the measured DOFs, which eliminates the remaining unmeasured DOFs to improve computational efficiency. However, there may still arise a numerically ill-conditioned problem, if the system condensation is not adequate for numerical programming or if the system condensation is not recalibrated with respect to the structural changes. This numerical problem is resolved in the present study by adopting more accurate accelerated improved reduced system (AIRS) as well as by updating the transformation matrix at every step. The criterion on the required accuracy of the condensation method is also proposed. Finally, numerical verification results of the present accelerated inverse perturbation method (AIPM) are presented.

Key Words : Damage Detection, Inverse Perturbation Method, System Condensation, Accelerated Improved Reduced System (AIRS), Accelerated Inverse Perturbation Method (AIPM)

1. Introduction

In recent years, structural damage detection has become a very important research issue in the areas of aerospace, naval, offshore, and nuclear engineering. The structural damage detection problem is very similar to the structural optimization problem from the mathematical point of view-they are all inverse problems. Many researchers have related the change in dynamic characteristics (natural frequencies and mode shapes) to the change in structural properties (stiffness and mass) and then have applied optimization theory to identify structural damages. Extensive reviews on the previous works in structural damage detection are provided by Doebling (1996) and Farrar (1997).

In the optimal matrix update method, the optimization theory is utilized to minimize the modal force errors while preserving the symmetry (Kammer, 1988), zero/nonzero pattern of system matrices (Smith and Beattie, 1991), or the load paths of a structure (Chen and Garba, 1988). However, these methods may smear the local structural changes throughout the entire structure and thus mislead damage locations. Thus, Kaouk and Zimmerman (1994) proposed a new approach by which the rank of structural damages can be minimized as an objective function. Unfortunately their approach is not practical because the number of da-

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mages needs to be known in advance in order to measure the rank of the damage (Doebling, 1996). As an alternative to the optimal matrix update method, Lee and Shin (2002) and Cho et el. (2002) have recently introduced the structural damage identification methods based on experimentally measured frequency response functions.

Recently, Choi (2001) adopted the inverse perturbation method to resolve troublesome issues that have arisen in the previous studies. He applied a nonlinear least squares method to detect structural damage, in which a greater number of constrained equations than unknown parameters was required for reliable results. If all unmeasured DOFs are treated as the unknown variables, a considerable computational effort will be required. Thus, he used a system condensation method to transform the unmeasured DOFs into measured DOFs, which eliminates the unmeasured DOFs to improve computational efficiency. However, there still remains a problem of numerical ill-conditioning. This problem may occur when the system condensation is not accurate enough for the least square method (Gafka and Zimmerman, 1996).

The purpose of the present study is to resolve numerically ill-condition problem by using more accurate accelerated improved reduced system (AIRS ; Kim and Kang, 2001) as well as updating the transformation matrix at every step to instantaneously reflect the latest structural change.

2. Inverse Perturbation Method

2.1 General formulation

Because the change in dynamic characteristics are related to the modifications of the structural properties in both the structural optimal design and structural damage identification, applications the nonlinear inverse perturbation method can be applied to the structural damage detection problems (Choi, 2001).

For an undamaged structure, the general form of the finite element eigenproblem can be derived in the form of

$$[k]\{\phi\}=\lambda[m]\{\phi\} \tag{1}$$

where $[k]$ and $[m]$ are the stiffness and mass matrices, respectively, and λ and $\{\phi\}$ represent an eigenpair. The structural damage may perturb the structure and the equilibrium equation for the perturbed system can be expressed as

$$[k']\{\phi'\}=\lambda[m']\{\phi'\} \tag{2}$$

The stiffness and mass matrices of the perturbed system can be represented as a sum of those for undamaged structure system and their damage-induced perturbations given by

$$\begin{aligned}[k'] &= [k]+[\Delta k] \\ [m'] &= [m]+[\Delta m]\end{aligned} \tag{3}$$

By using Eq. (3), Eq. (2) can be rewritten as

$$([k]+[\Delta k])\{\phi'\}=\lambda'([m]+[\Delta m])\{\phi'\} \tag{4}$$

2.2 Parameterization in structural and modal changes

There are two kinds of unknowns in the inverse perturbation problems : Structural parameters and response parameters. The former is related to physical parameters and the latter represents the unspecified (unmeasured) DOFs.

The structural changes can be decomposed into J finite-element changes as

$$[\Delta k]=\sum_{e=1}^{J}[\Delta k_e] \quad [\Delta m]=\sum_{e=1}^{J}[\Delta m_e] \tag{5}$$

where

$$[\Delta k_e]=[s^k(\alpha_e)] \quad [\Delta m_e]=[s^m(\alpha_e)] \tag{6}$$

$$\alpha_e \equiv \Delta p_e/p_e \tag{7}$$

The non-dimensional variable α_e represents the change in a physical parameter, and zero value means no damage-induced change in the physical parameter. To improve numerical stability, it is preferable to express them by the ratio of the parameter change to the original value. Furthermore, each element change can be expressed as a sum of fractional changes of each physical parameter, such as elastic stiffness and second moment of area (change of thickness or width). This relationship may be linear or nonlinear, depending on the property of physical parameters. Thus, Eq. (6) can be expressed as

$$[S^k(\alpha_e)]=\frac{\partial[k]}{\partial\alpha_e}\alpha_e+\frac{1}{2!}\frac{\partial^2[k]}{\partial\alpha_e^2}\alpha_e^2+\cdots \quad (8)$$

$$[S^m(\alpha_e)]=\frac{\partial[m]}{\partial\alpha_e}\alpha_e+\frac{1}{2!}\frac{\partial^2[m]}{\partial\alpha_e^2}\alpha_e^2+\cdots \quad (9)$$

The response parameters denote the natural frequencies (eigenvalues) and mode shapes (eigenvectors). In general, measuring the changes in the frequencies are relatively easier than measuring the changes in the mode shapes. It is neither possible nor desirable to measure all DOFs of each mode shape, especially for large structures. Because only some DOFs can be measured, the rest must be determined such that they satisfy new equilibrium of the damaged structure. The unmeasured DOFs can be considered as the additional unknown variables, called the response or characteristic parameters.

$$[k]\{\phi'\}=[k_p \quad k_s]\begin{Bmatrix}\phi_p'\\ \phi_s'\end{Bmatrix} \quad (10)$$

In Eq. (10), the subscripts p and s indicate the primary (or measured) DOFs and the secondary (or unmeasured) DOFs, respectively.

2.3 Inverse perturbation method

Because the exact structural changes are not known and large part of DOFs cannot be measured in advance, Eq. (1) cannot be satisfied due to incompletely measured data.

Then, one may compute the residual error as

$$\{R\}\equiv[k']\{\phi'\}-\lambda'[m']\{\phi'\} \quad (11)$$

This residual error should vanish at the new equilibrium of the perturbed system. This is only a necessary condition for satisfying the new equilibrium. When more than one mode is used, the residual errors can be expressed as

$$[R_L]\equiv[k'][\Phi_L']-[m'][\Phi_L'][\Lambda_L'] \quad (12)$$

where L is the number of modes used in the inverse perturbation. The dimension of the matrix R_L is n by L. The number of equations used in numerical programming is

$$\textit{The number of equations}=n\times L \quad (13)$$

In structural optimization problems, structural changes $[\Delta k]$ and $[\Delta m]$ are determined so as to satisfy desired characteristic changes $[\Delta\phi]$ and $[\Delta\lambda]$. Because there may exist many feasible designs, the best design is determined by minimizing the objective function such as minimum weight or minimum change. As the structural optimization problem is an underdetermined problem, it usually needs fewer equations than unknowns. However, for unique damage, the structural changes determined should be unique to the given structural damage detection problem. This requires more equations than unknowns. The number of unknown parameters is

$$\textit{The number of unknowns}=\underbrace{n_s\times L}_{\substack{response\\parameters}}+\underbrace{n_{\alpha_e}}_{\substack{structural\\parameters}} \quad (14)$$

where n_s and $n_{\alpha e}$ are the number of ϕ_s and structural parameters, respectively. The number of modes to be used for damage detection must be determined such that the number of equations given by Eq. (13) is larger than the number of unknowns given by Eq. (14).

Substituting Eqs. (3) and (5-10) into Eq. (11) gives

$$\begin{aligned}\{R\}&=([k]-\lambda'[m])\{\phi'\}+([\Delta k]-\lambda'[\Delta m])\{\phi'\}\\&=([k_p]-\lambda'[m_p])\{\phi_p'\}+([k_s]-\lambda'[m_s])\{\phi_s'\}\\&\quad+\sum_{e=1}^{s}([S_p^k(\alpha_e)]-\lambda'[S_p^m(\alpha_e)])\{\phi_p'\}\\&\quad+\sum_{e=1}^{s}([S_s^k(\alpha_e)]-\lambda'[S_s^m(\alpha_e)]\{\phi_s'\})\end{aligned} \quad (15)$$

2.4 Over-determined nonlinear problem

The least squares method can be applied to the over-determined problem. In general, a regression that perfectly fits correct solution does not exist. However, the residual errors given by Eq. (15) can be minimized as

$$Min\left(\sum_{j=1}^{L}\{R\}_j^T\{R\}_j\right) \quad (1)$$

For improved numerical behavior, the normalization procedure can be used as follows:

$$Min\left(\sum_{j=1}^{L}\{R\}_j^T[W]_j^{(k)}\{R\}_j\right) \quad (17)$$

where $[W]_j^{(k)}$ is the weighting matrix for the j-th mode and k-th iteration. The weighting matrix can be determined by the gradient of residual vector as (Luenberger, 1984)

$$[W]_j^{(k)}=([\nabla R_j(X_j^{(k)})][\nabla R_j(X_j^{(k)})]^T)^{-1} \quad (18)$$

where $X_j^{(k)}$ is the vector of unknown variables, which consists of structural parameters and the j-th response parameters. As the computation of weighting matrix $[W]_j^{(k)}$ at each step is expected to be quite expensive, the weighting matrix can be approximated as a diagonal matrix by using a new mode shape $\{\phi'\}_j$. The elements along the diagonal are then given by

$$W_{ii}=(\phi'_{ij})^2 \quad (19)$$

where the subscript i represents the components of each DOF consisting of a mode vector. After the normalization procedure, the minimization of residual forces is now just like the minimization of the residual energy error at each DOF.

3. Accelerated Inverse Perturbation Method

In practice, all DOFs cannot be measured and thus the unmeasured DOFs must be considered as the unknown variables in the damage detection procedure, which may degrade the computational efficiency. Thus, a system condensation method will be used to transform the unmeasured DOFs into the measured DOFs. This may improve the computation speed. However, one need to pay special attention to the high sensitivity of the nonlinear least squares method to the transformation errors of the system condensation methods.

3.1 System condensation

The accuracy of the solutions obtained by system condensation depends on the selection of primary DOFs. It has been known that, even though a good selection of primary DOFs has been made, only one third of the eigenvalues calculated in the reduced subspace are reliable.

A general eigenproblem can be expressed in partitioned form as

$$\begin{bmatrix} k_{pp} & k_{ps} \\ k_{sp} & k_{ss} \end{bmatrix}\begin{Bmatrix} \phi_p \\ \phi_s \end{Bmatrix}=\lambda\begin{bmatrix} m_{pp} & m_{ps} \\ m_{sp} & m_{ss} \end{bmatrix}\begin{Bmatrix} \phi_p \\ \phi_s \end{Bmatrix} \quad (20)$$

The secondary DOFs $\{\phi_s\}$ are the DOFs to be condensed out. The relation between the primary and secondary sets is obtained from Eq. (20) as

$$\begin{aligned}\{\phi_s\}&=-([k_{ss}]-\lambda[m_{ss}])^{-1}([k_{sp}]-\lambda[m_{sp}])\{\phi_p\} \\ &\equiv[T(\lambda)]\{\phi_p\}\end{aligned} \quad (21)$$

where the transformation matrix $[T(\lambda)]$ is frequency-dependent.

Increasing attention has been given to the improvement of the transformation step, which is the main source of error in condensation. The matrix inversion in Eq. (21), containing unknown eigenvalues, can be approximated as an infinite series given by

$$\begin{aligned}&([k_{ss}]-\lambda[m_{ss}])^{-1} \\ &=([I]+\lambda[A_s]+\lambda^2[A_s]^2+\cdots)[k_{ss}]^{-1}\end{aligned} \quad (22)$$

where

$$[A_s]=[k_{ss}]^{-1}[m_{ss}] \quad (23)$$

For simplicity, higher-order terms are neglected. In Guyan's static condensation, the mass associated with the secondary set is neglected, and the transformation becomes constant for all modes (Guyan, 1965).

$$\{\phi_s\}\cong-[k_{ss}]^{-1}[k_{sp}]\{\phi_p\}\equiv[T_o]\{\phi_p\} \quad (24)$$

$$\{\phi\}_{app}=\begin{Bmatrix} \phi_p \\ \phi_s \end{Bmatrix}_{app}\cong\begin{bmatrix} I \\ T_o \end{bmatrix}\{\phi_p\}\equiv[T_G]\{\phi_p\} \quad (25)$$

Equation (25) implies that the original n-dimensional system can be reduced by a linear combination of p bases that are column vectors of $[T_G]$. By applying Eq. (25), Eq. (20) can be reduced in the form of

$$[K_G]\{\phi_p\}=\lambda_r[M_G]\{\phi_p\} \quad (26)$$

where

$$\begin{aligned}[K_G]&=[T_G]^T[K][T_G] \\ [M_G]&=[T_G]^T[M][T_G]\end{aligned} \quad (27)$$

From Eq. (26), one may write the relation as

$$\lambda\{\phi_p\}\cong\lambda_r\{\phi_p\}=([M_G]^{-1}[K_G])\{\phi_p\} \quad (28)$$

Substituting Eqs. (23)~(28) into Eq. (22) yields

$$\begin{aligned}&\{\phi_s\}_{app} \\ &=([T_o]+[T_1]+[T_2]+[T_3]+\cdots)\{\phi_p\}_{app}\end{aligned} \quad (29)$$

$$\begin{aligned}[T_o]&=-[k_{ss}]^{-1}[k_{sp}] \\ [T_1]&=[k_{ss}]^{-1}([m_{sp}]+[m_{ss}][T_o])([M_G]^{-1}[K_G]) \\ [T_2]&=[A_s][T_1]([M_G]^{-1}[K_G]) \\ [T_3]&=[A_s][T_2]([M_G]^{-1}[K_G])\end{aligned} \quad (30)$$

which is the AIRS (Kim and Kang, 2001). It is a well known fact that the series converges in lower modes whenever λ_r is smaller than λ_s which is the lowest eigenvalue evaluated with $[k_{ss}]$ and $[m_{ss}]$.

Considering more terms in Eq. (29) may provide further improved results, possibly with increased computational cost. However, through some numerical examples, considering the terms up to the third order expansion is found to be sufficient, which is called AIRS3. On the other hand, the improved reduced system (IRS, O'Callahan, 1989) is based on the first-order approximation and Gafka and Zimmerman (1996) used it to condense a damaged system.

$$\{\phi\}_{app}=\left([T_G]+\begin{bmatrix}0 & 0\\ 0 & k_{ss}^{-1}\end{bmatrix}[m][T_G][D_G]\right)\{\phi_p\} \\ =[T_{IRS}]\{\phi_p\} \tag{31}$$

$$[T_{IRS}]=\begin{bmatrix} I \\ T_o+T_1\end{bmatrix}=\begin{bmatrix} I \\ T_{AIRS1}\end{bmatrix} \tag{32}$$

The transformation errors by various condensation methods such as Guyan's reduction, IRS, and AIRS3 are compared in Figure 1. The transformation errors are for the vertical displacement at node 14 of the numerical example beam. The transformation error is defined as the absolute value of the difference between the exact and transformed secondary DOFs.

$$\{E\}=|\{\phi_s\}-\{\phi_s\}_{app}| \tag{33}$$

Figure 1 shows that the transformation method requiring higher modes has relatively large error. This is why only lower modes are considered as

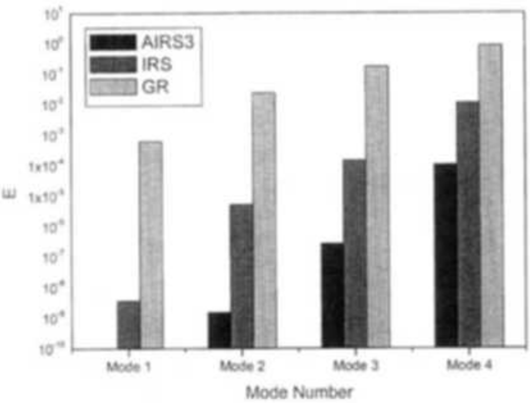

Fig. 1 Transformation errors E for the translational DOF at node 14

Table 1 Transformation errors for the representative four DOFs

Mode	DOF	With Condensation			Without Condensation
		GR (Guyan, 1965)	IRS (O'Callahan, 1989)	AIRS3 (Present)	Exact
1	1T	0.001537218	0.001537198	0.001537198	0.001537198
	6R	0.000577075	0.000577069	0.000577069	0.000577069
	14T	0.249507234	0.249508750	0.249508750	0.249508750
	14R	0.001109933	0.001109885	0.001109885	0.001109885
2	1T	−0.011812130	−0.011807422	−0.011807419	−0.011807419
	6R	−0.003083731	−0.003082264	−0.003082263	−0.003082263
	14T	−0.939882780	−0.940101181	−0.940101229	−0.940101229
	14R	−0.001398256	−0.001391191	−0.001391189	−0.001391189
3	1T	0.042238820	0.042130681	0.042130466	0.042130465
	6R	0.006700199	0.006665392	0.006665379	0.006665379
	14T	0.949319552	0.950915250	0.950916611	0.950916613
	14R	−0.007286399	−0.007340974	−0.007341014	−0.007341014
4	1T	−0.118102307	−0.117085994	−0.117080784	−0.117080845
	6R	−0.007465129	−0.007139152	−0.007138988	−0.007138998
	14T	0.829672351	0.836378078	0.836465661	0.836466512
	14R	0.018777605	0.018559340	0.018556390	0.018556360

(T=translational DOF, R=rotational DOF)

accurately as possible in most condensation methods. Thus, criterion on the bound of modes should be provided for each condensation method. For the same example, the transformation errors for some specific DOFs are shown in Table 1. Here, the term exact means that the mode shapes are evaluated by using full DOFs, without condensation.

3.2 Inverse perturbation method using AIRS

It is important to point out that the system transformation and inverse perturbation methods are coupled. In damage detection, the measured data of primary DOFs are substituted into the equilibrium equation and the unmeasured secondary DOFs are obtained as

$$\begin{aligned}\{\phi_s'\} &= -([k_{ss}'] - \lambda'[m_{ss}'])^{-1}([k_{sp}'] - \lambda'[m_{sp}'])\{\phi_p'\} \\ &\equiv [T'(\lambda', \alpha_e)]\{\phi_p'\}\end{aligned} \tag{34}$$

However, the transformation matrix used in the inverse perturbation method must be updated so that the structural changes can be instantaneously reflected in the analysis procedure.

The transformation matrix in Eq. (34) can be simplified by using AIRS as a function of structural parameters only.

$$[T'(\lambda', \alpha_e)] \cong [T'_{AIRS}(\alpha_e)] \tag{35}$$

As a result, the secondary DOFs can be replaced with the primary DOFs and the response parameters can be successfully eliminated in the inverse perturbation method. The final form of the residual error using system transformation is given by

$$\begin{aligned}\{R\} &= ([k] - \lambda'[m])\{\phi'\} + ([\Delta k] - \lambda'[\Delta m])\{\phi'\} \\ &\cong \left([k] - \lambda'[m] + \sum_{e=1}^{L}([S^k(\alpha_e)] - \lambda'[S^m(\alpha_e)])\right)\begin{bmatrix} I \\ T'_{AIRS} \end{bmatrix}\{\phi_p'\}\end{aligned} \tag{36}$$

How many terms should be used for the transformation matrix $[T'_{AIRS}]$ is not yet determined, and there is a question on the criterion for determining the accuracy of a condensation method that is appropriate for an inverse perturbation method. Thus, based on the error tolerance of the nonlinear least squares method, a simplified criterion will be proposed as follows: Even for a structure without any change, there may exist some residual error due to the transformation error in the condensation method. Thus, the norm of the residual vector $\{R\}$ should be smaller than a pre-specified tolerance (ε) as

$$\|\{R\}\| \leq \varepsilon \tag{37}$$

Figure 2 shows the effects of random noise on the residual error. The random noise represents the transformation errors in the condensation method. In Fig. 2, the term "exact" represents the residual error for the case of zero random noise, computed using a 64-bit double precision. The residual error can be considered as the numerical error. It is numerically observed that the residual error of Eq. (36) increases as the magnitude the random noise increases. The expected magnitude the random noise for the tolerance level $\varepsilon=0.1$ is on the order of 10^{-8}, which can serve as a guide for the choice of the condensation method. Because considering ever higher modes tends to increase the condensation error, one must confirm if the mode satisfies the criterion of Eq. (37). Figure 2 shows that AIRS3 satisfies the above tolerance condition up to the third mode, while IRS(AIRS1) satisfies the above tolerance condition only for the first mode. In general, the order of AIRS must be determined by the number of modes to be considered. If higher modes are required in the inverse perturbation method, the order of AIRS must be increased. Table 2 shows the residual errors produced by the transformation errors during system condensation.

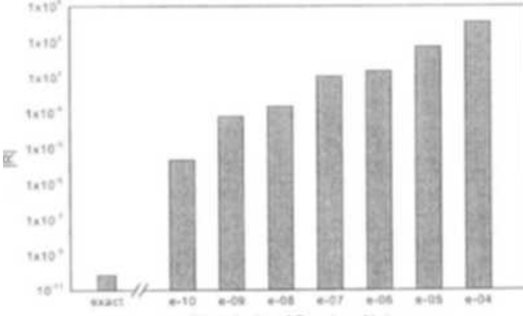

Fig. 2 Effects of random noise on the residual error |R|

Table 2 Residual errors due to the transformation errors in system condensation

Mode	DOF	With Condensation			Without Condensation
		GR (Guyan, 1965)	IRS (O'Callahan, 1989)	AIRS3 (Present)	Exact
1	1T	−0.015705895	8.44703E-07	1.47755E-09	1.31293E-09
	6R	−0.325633813	1.44844E-06	1.44355E-08	9.66247E-09
	14T	−2.571418012	−6.64375E-06	−3.53248E-09	−1.51704E-09
	14R	−0.632357653	2.38446E-05	8.99599E-08	−2.53785E-08
2	1T	4.429924530	−0.004756375	−2.70177E-06	1.67100E-09
	6R	62.54439481	−0.022434156	−2.83122E-07	−2.79397E-08
	14T	339.7999582	0.036873560	6.19004E-06	−2.86382E-08
	14R	29.12511988	−0.117752351	−3.09274E-05	2.04891E-07
3	1T	−133.2319953	0.391336474	0.000585497	3.43539E-09
	6R	−1136.622160	6.041255075	0.002298595	−5.19678E-07
	14T	−2748.527803	−1.639287232	−0.002613661	−2.16533E-08
	14R	1222.612315	5.512731884	0.004087100	−5.02914E-08
4	1T	1247.412397	−9.790983759	0.065582307	1.10100E-08
	6R	3719.432169	−210.9049270	−0.623880744	−1.49012E-08
	14T	−8999.167615	−27.55777317	−0.191453247	−2.32831E-08
	14R	−9738.002602	165.2275536	2.103506967	5.21541E-08

(T=translational DOF, R=rotational DOF)

4. Numerical Example

For a numerical example, a uniform cantilever beam is considered for structural damage identification. The beam has the cross sectional area $A=40$ mm × 40 mm, length $L=1000$ mm, Young's modulus $E=6.9\times10^4$ MPa, and the mass density $\rho=2.7\times10^{-9}$ Ns^2/mm^4. The beam is assumed to have the planar transverse motion only. Figure 3 shows the finite element model for the beam, having 15 small elements and 10 large elements (Papadopolous, 1998) with a total of fifty DOFs. For the numerical simulation on damage identification, four defects are placed on the 5th, 10th, 19th and 25th elements by reducing the thickness h_e by 20%, 25%, 25%, and 30%, respectively, as shown in Fig. 4.

The change in the second moment of area, ΔI_e, can be expressed as

$$\Delta I_e = I_e(3\alpha_e + 3\alpha_e^2 + \alpha_e^3) \tag{38}$$

where I_e is the second moment of area before the damage and α_e is the structural parameter

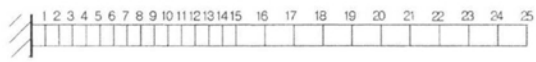

Fig. 3 Numerical example : 25 finite elements model of a cantilevered beam

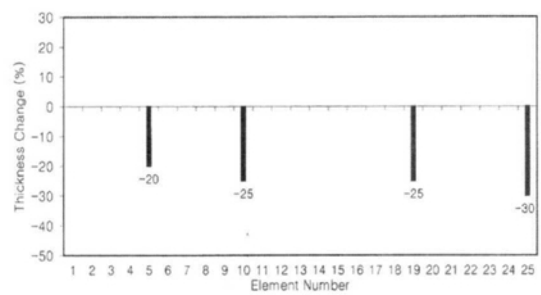

Fig. 4 Pre-specified locations and magnitudes of the changes in beam thickness

indicating the extent of the thickness reduction due to the damage. They are defined by

$$I_e = \frac{bh_e^3}{12},\quad \alpha_e = \frac{\Delta h_e}{h_e} \tag{39}$$

The changes in the element stiffness and mass matrices can be expressed as

Table 3 Performance of the present method for the example problem given in Fig. 3

	Inverse Perturbation Method (Choi, 2001)	Accelerated Inverse Perturbation Method (Present)
Number of unknowns	305	25
Required uumber of modes	7	3
Computation time (seconds)	3270	470

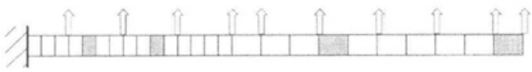

Fig. 5 Placement of ten sensors by using the successive elimination method

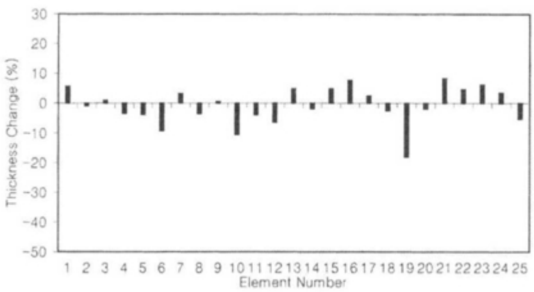

Fig. 6 Damage detection by using fundamental mode and ten measurement points

$$[\Delta k]_e=[k]_e\Delta I_e=[k]_e(3\alpha_e+3\alpha_e^2+\alpha_e^3)$$
$$[\Delta m]_e=[m]_e\frac{\Delta h_e}{h_e}=[m]_e\alpha_e \quad (40)$$

Substituting Eq. (41) into Eq. (37) yields

$$\{R\}\cong\left([k]-\lambda'[m]+\sum_{e=1}^{L}([k]_e(3\alpha_e+3\alpha_e^2+\alpha_e^3)-\lambda'[m]_e\alpha_e)\right)\begin{bmatrix} I \\ T'_{AIRS3}\end{bmatrix}\{\phi_p'\} \quad (41)$$

To identify the pre-specified defects in Fig. 4, we can acquire the DOFs of several changed mode shapes. In this study, ten translational DOFs are selected by using the successive elimination (Shah and Raemund, 1982; Penny et al., 1994), and three mode shapes are used. The fourth mode is excluded because it is found to violate the condition of Eq. (38) when the tolerance is 0.1.

Fig. 6 shows the damage identification obtained by using the first mode only. Though the residual could be minimized to satisfy the tolerance, the result is not satisfactory as insufficient number of equations are used. Thus, it is required

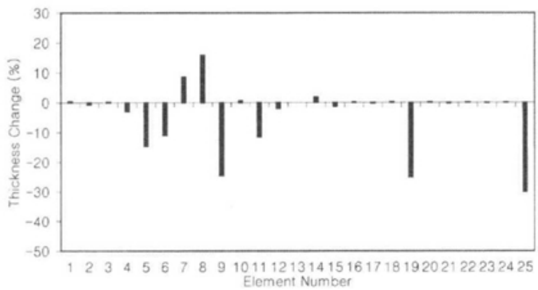

Fig. 7 Damage detection by using the lowest two modes and ten measurement points

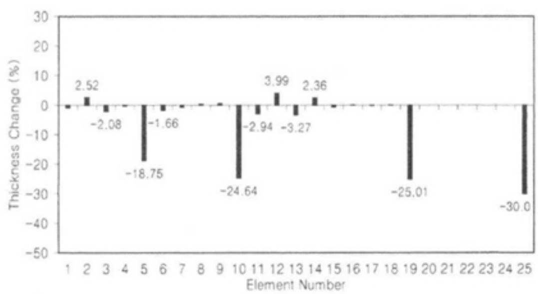

Fig. 8 Damage detection by using the lowest three modes and ten measurement points

to use additional higher modes to obtain more reliable results. Figures (7-8) show that further improved results can be obtained by using more higher modes. Figure 8 clearly shows that the four pre-specified damages could be identified almost accurately when the lowest three modes are used. Some unwanted noise is inevitable for most error minimizing methods. Using higher modes may reduce the distributed noise, possibly with increased computational efforts required to compute over the third order of AIRS. Thus, there should be a compromise between the accuracy and computational efficiency. Table 3 compares the computational efficiencies of the present method and the original inverse perturbation method.

5. Conclusions

The inverse perturbation method has been applied to structural damage detection. Because all DOFs cannot be measured in practice, the rest of the unmeasured DOFs should be considered as the unknowns, which may degrade the computational efficiency of the damage identification analysis. Thus, the present study proposes the accelerated inverse perturbation method (AIPM) for structural damage detection, in which the AIRS is used to transform the unmeasured DOFs into the measured DOFs. A criterion on the required accuracy of the condensation method to be chosen is also proposed. It is shown that the criterion could depend on the tolerance of the numerical method and the number of modes used in the solution process. In addition, it is numerically shown that the order of AIRS can be determined by applying the criterion proposed in this study. Finally, a numerical verification of the present AIPM is conducted. The rank deficiency, which is another unsolved numerical problem, will be the research issue in the on-going study.

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